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Assignment 5

Q1 [Goldenfeld 6-1 p185]

Gaussian fluctuations for $T<T_c$. Let us use our class notation; $\eta(r) = \langle\eta\rangle + \epsilon(r)$

for $T<T_c$, $\langle\eta\rangle = \sqrt{\frac{-at}{b}}$

$$L = \int d^d r \left[at\eta^2 + \frac{b}{2}\eta^4 + \frac{\gamma}{2}(\nabla\eta)^2\right]$$

$$at\eta^2 = at[\langle\eta\rangle+\epsilon]^2 = at[\langle\eta\rangle^2 + \epsilon^2 + \underbrace{2\epsilon\langle\eta\rangle}] \quad -(1)$$

$$\frac{b}{2}\eta^4 = \frac{b}{2}[\langle\eta\rangle^2 + \epsilon^2 + 2\epsilon\langle\eta\rangle][\langle\eta\rangle^2 + \epsilon^2 + 2\epsilon\langle\eta\rangle]$$

$$= \frac{b}{2}[\langle\eta\rangle^4 + \langle\eta\rangle^2\epsilon^2 + 2\epsilon\langle\eta\rangle^3 + \epsilon^2\langle\eta\rangle^2 + \epsilon^4 + \cancel{2\epsilon^3\langle\eta\rangle} + 2\epsilon\langle\eta\rangle^3 + \cancel{2\epsilon^3\langle\eta\rangle} + 4\epsilon^2\langle\eta\rangle^2]$$

throw away ϵ^3 terms [Gaussian approx.] & ϵ^4

$$\frac{b}{2}\eta^4 \simeq \frac{b}{2}[\langle\eta\rangle^4 + 6\langle\eta\rangle^2\epsilon^2 + 4\epsilon\langle\eta\rangle^3 + \cancel{\epsilon^4}]$$

$$= \frac{b}{2}\langle\eta\rangle^4 + 3b\langle\eta\rangle^2\epsilon^2 + \underbrace{4\epsilon\langle\eta\rangle\left[\frac{-at}{b}\right]\cdot\frac{b}{2}} + \cancel{\epsilon^4\frac{b}{2}} \quad (2)$$

Gaussian approx.

Note that underlined terms in eq (1) & (2) cancel each other

$$\Rightarrow L = \beta\underbrace{L[\langle\eta\rangle]}_{\text{M.F.}} + \int d^d r\left[\frac{\gamma}{2}(\nabla\epsilon)^2 + at\epsilon^2 + 3b\langle\eta\rangle^2\epsilon^2 + \frac{b}{2}\cancel{\epsilon^4}\right]$$

Also $3b\langle\eta\rangle^2 = 3b\left[\frac{-at}{b}\right] = -3at$

Hence

$$L = L[\langle\eta\rangle] + \int d^d r \left[\frac{\gamma}{2} (\nabla\epsilon)^2 - 2at\epsilon^2 \right), \quad t<0$$

$$= L(\langle\eta\rangle) + \int d^d r \left\{ \frac{\gamma}{2} [\nabla\epsilon(x)]^2 + 2a|t|\epsilon^2(x) \right\}$$

So we the same form for the Gaussian fluctuations contribution to the free energy, except for an additional factor of 2. $at \longrightarrow 2a|t|$

The correlation length will be similarly defined with an additional factor of 2.

Rest of the question is straightforward.